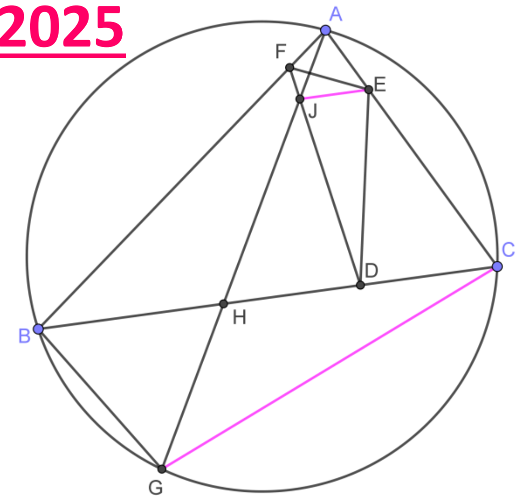
Author's Solution- Aug-2025



Construction :

Let FD & AG intersect at J. Join JE & GC.

Proof:

ΔDEF is orthic triangle

$\Longrightarrow$ ABDE, AFDC & BFEC are concyclic.

$\Longrightarrow \angle B = \angle AEF = \angle CED$ -----------------------------------(1)

$\Longrightarrow \angle A = \angle EDC = \angle FDB$ -----------------------------------(2)

$\Longrightarrow \angle C = \angle AFE = \angle BFD$ ---------------------------------- (3)

In Δs JHD & ΔECD, $\angle ECD = \angle JHD$ ----------------- (4) [AC = AH given]

$\angle JDH = \angle EDC$ [from(2)]

$\Longrightarrow$ $\angle$DJH = $\angle DEC$ --- (5)

$\Longrightarrow$ $\angle DJH$= $\angle$AJF = $\angle AEF$ [from (1) & (5)]

$\Longrightarrow$ $AFJE$ is concyclic -------------------------------------- (6)

from (4)

$\angle HAC = 180° - 2\angle C$ [from (2)]

$\angle AFE = \angle AJE = \angle C$ [from (3) & (6)]

(3) & (6) $\rightarrow$ $\angle AJE = \angle AEJ = \angle C$ ---(7)

(5) & (7) $\rightarrow$ $\angle DJE = \angle DEJ$

$\Longrightarrow DJ = DE$ --(8)

In ΔBGC & ΔFJE

$\angle$BGC = $\angle FJE$ (both are 180-$\angle BAC$)

$\angle BCG$ = $\angle FEJ$ (both are equal to $\angle EAJ$)

$\Longrightarrow$ ΔBGC & ΔFJE are similar

$\Longrightarrow \frac{BG}{FJ} = \frac{BC}{EF}$

BG x EF = BC x FJ

$\Longrightarrow$ BG x EF = BC (DF - DJ)

$\Longrightarrow$ BG x EF = BC (DF - DE) [DE = DJ (from (8)]

BC(DF - DE) = BG x FE --------------------------------Proved

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